Temperature and Pressure Discontinuity Stresses in Compact Heat Exchangers

G. M. Swisher*
Wright State University, Dayton, Ohio

IN an era of increased material strengths and supersonic aircraft, the weight of an aircraft heat exchanger is coming under close scrutiny. Many aircraft parts are being optimized using a performance criteria of minimizing the strength to weight ratio. Gardner[1] is the first published work which calculates the stress in a heat exchanger tube sheet by treating the tubes as an elastic foundation on which the tube plate rests. His analysis does not consider the following: 1) thermal stresses may arise due to the temperature differential between tube sheet forces; 2) discontinuity stresses may be significant between the tube sheet and the outer shell. K. A. G. Miller[2] extends the work of Gardner by accounting for the weakening effect of the holes in the plate by introducing expressions for the deflection and ligament efficiencies defined by Gardner. These efficiencies are shown to lie within the range specified by Gardner and this leads to definite design equations. Figure 1 shows free body diagrams of the heat exchanger tube sheet and the shell. Consider the shell and tube sheet with an applied shear V_0 and an applied moment M_0 with positive directions as shown. M_0 has the dimensions of in.-lbf/in. of circumference and V_0 has the dimensions of lbf/in. of circumference. Roark[3] has compiled an extensive table of deformations and stresses for pressure vessels, shells, and plates. These formulae can be used with the theories of superposition and discontinuity stresses to predict the stress level at the change of shape (shell and tube sheet connection).

If the material does not fracture the deformations and slopes of the tube sheet and shell must be identical. Referring to Fig. 1, V_0 acting on the shell causes a radial shrinking $\delta_3 = V_0/2D\lambda_1^3$; V_0 acting on the tube sheet causes a radial elongation $\delta_1 = V_0a(1 - \mu)/Eh$; M_0 acting on the shell causes an elongation $\delta_4 = M_0/2D\lambda_1^2$; M_0 acting on the tube sheet causes an elongation $\delta_2 = 6aM_0/Eh^2$. A temperature differential ΔT across the tube sheet with the highest temperature on face A causes a tube sheet radial shrinking of $a\alpha\Delta T/2$. The shell pressure causes a radial shell elongation of $a(\sigma_h - \mu\sigma_l)/E$ where $a \triangleq$ tube sheet and shell radius, $D \triangleq$ flexural stiffness of shell, $E \triangleq$ modulus of elasticity, $h \triangleq$ tube sheet thickness, $\mu \triangleq$ Poisson's ratio, $\sigma_h \triangleq$ shell hoop stress due to pressure, $\sigma_l \triangleq$ shell longitudinal stress due to pressure, $\alpha \triangleq$ temperature coefficient of expansion, $t_s \triangleq$ shell thickness, and $\lambda_1 \triangleq [3(1 - \mu^2)/t_s^2a^2]^{1/4}$. The radial deflection equation becomes

$$\underbrace{-\alpha a\Delta T/2 + \delta_1 + \delta_2}_{\text{tube sheet radial elongation}} = \underbrace{a(\sigma_h - \mu\sigma_l)/E + \delta_4 - \delta_3}_{\text{shell radial elongation}} \quad (1)$$

Received July 23, 1970. This work was sponsored by United Aircraft Products Inc., Dayton, Ohio.
* Assistant Professor of Engineering.

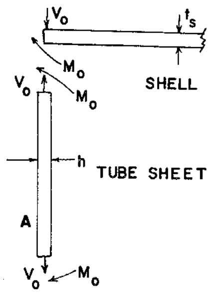

Fig. 1 Free body diagrams of tube sheet and shell.

In practice δ_1 and δ_2 are small compared to δ_3 and δ_4 and can be neglected. Matching the slopes of the tube sheet and the shell results in Eq. (2)

$$V_0/2D\lambda_1^2 - M_0/\lambda_1 D = a\alpha\Delta T/h \quad (2)$$

Upon solving Eqs. (1) and (2) for M_0 and V_0 as a function of ΔT the states of stress of the shell and the tube sheet can be calculated using superposition. The total stresses in the shell, in practice, have been the limiting stresses. A digital computer program was written to solve for the total shell stresses.

The longitudinal stress due to M_0 is given by $6M_0/t_s^2$. The shear stress is given by V_0/t_s. The longitudinal stress due to pressure is given by $Pa/2t_s$. The hoop stress due to pressure is given by Pa/t_s. The direct hoop stress due to V_0 is given by $-2V_0a\lambda_1/t_s$. The direct hoop stress due to M_0 is given by $2M_0\lambda_1^2a/t_s$. The hoop bending stress due to M_0 is given by $\mu 6M_0/t_s^2$.

The hoop stresses at the outer and inner surfaces can now be calculated. The outer surface hoop stress is given by $-2V_0a\lambda_1/t_s + 2M_0\lambda_1^2a/t_s - \mu 6M_0/t_s^2 + Pa/t_s$. The inner surface hoop stress is given by $-2V_0a\lambda_1/t_s + 2M_0\lambda_1^2a/t_s + \mu 6M_0/t_s^2 + Pa/t_s$.

The longitudinal stress at the outer surface equals $Pa/2t_s - 6M_0/t_s^2$. The longitudinal stress at the inner surface equals $Pa/2t_s + 6M_0/t_s^2$.

The tube sheet stresses, in practice, are limited by the pressure stresses obtained by an analysis such as that of Gardner.[1] His pressure stresses must have superimposed upon them the stresses due to V_0 and M_0 although these stresses will generally be small compared with the pressure stresses.

The author concluded that the optimum design of a compact heat exchanger can be arrived at by a "brute" force method of attack since the total failure stresses can now be calculated using the previous method. Further work remains to be done in selecting optimum configurations with respect to weight.

References

[1] Gardner, K. A., "Heat Exchanger Tube-Sheet Design," *Journal of Applied Mechanics*, Vol. 15, Dec. 1948, pp. 377–385.

[2] Miller, K. A. G., "The Design of Tube Plates in Heat Exchangers," *Transactions of the Institute of Mechanical Engineers*, 1951, pp. 215–227.

[3] Roark, R. J., *Formulas for Stress and Strain*, McGraw-Hill, New York, 1965.